# Scaling behavior in a proportional voting process

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We perform a statistical analysis on the proportional elections held in Brazil in October 1998. We show that the distribution of votes among candidates for the whole country follows a power law $N(v) \propto v^{-\alpha}$, with $\alpha = 1.00 \pm 0.02$, extending over two orders of magnitude. The voting distributions for several states of the federation also display scale-invariant behavior with $\alpha \approx 1$. We argue that this particular voting system can be modeled as a typical multiplicative process in which the choice of the candidate is governed by a product of probabilities. [S1063-651X(99)03907-0]

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The ubiquitous occurrence of ''scale-free'' phenomena has been the subject of intense scientific research in the fields of physics [1], geology [2], biology [3–5], and economics [6,7]. The absence of a characteristic scale in such diverse systems indicates that a power-law distribution should be representative of their statistics. More recently, scale-invariant correlations have also been detected in the social sciences. For instance, in the study of urban growth dynamics, long-range power-law correlations have been successfully used to model the morphology and the area distribution of systems of cities as well as the irregular scaling of individual urban perimeters [8,9].

Elections are fundamental social processes in democratic societies. The vote is one of the most effective instruments that regular citizens have to bring about changes in their communities. Consequently, it is not surprising that the ways in which people make their choices have been the subject of numerous studies on social behavior (see [10,11] and references therein). In contrast to these previous studies on the subject, the analysis presented here is not concerned with a *majority* voting process. In this paper, we show that the data obtained from the 1998 *proportional* election in Brazil display statistical features of a scale-invariant phenomenon. The observed uniformity in the statistical distribution for different states with large social and economic discrepancies is discussed. Finally, we suggest that a multiplicative process should be the rationale behind the voter's choice of a candidate in a proportional voting system.

On 4 October 1998, Brazil held general elections with compulsory voting. 106 101 067 electors from all states in the country voted to choose a president, 81 senators, and 513 congressmen. Also, voters from each state chose a governor and local representatives (state deputies). 57.6% of the ballots were collected by electronic voting machines, while the remainder were cast in traditional ballot boxes. Results from each state were made available through the web site of the Brazilian Federal Electoral Court [12]. We consider the results from each state for the positions of the local state deputies. This group comprises a number of candidates in each state that is significantly larger than the number of candidates competing for seats in the National Congress (federal deputies). For each state, we first normalize the votes of each candidate by the total number of voters. We then group in a histogram the number of candidates $N$ which received a certain fraction of votes, $v$.

In Fig. 1 we show the logarithmic plot of the voting distribution for the state of São Paulo, where the number of voters and candidates constitutes the largest electoral group in Brazil. In 1998, São Paulo had 1260 candidates for local representation and 23 321 034 voters. This number represents 21.99% of the voting population in the whole country. As can be seen, the number of candidates $N$ follows a power law $N(v) \propto v^{-\alpha}$, with $\alpha = 1.03 \pm 0.03$, extending over two orders of magnitude. We observe a similar behavior for other Brazilian states with a number of voters and candidates sufficiently large to allow for the same statistical treatment.

Next, we report and analyze results from the nationwide voting totals for local representation. The total number of candidates for state deputy in the country, that is, the sum of all candidates by state, was 10 535. Again, we normalize the votes of each candidate by the number of voters in their respective state. Subsequently, we rank all candidates by their normalized number of votes to perform the statistics for the whole country. This is justified by the aforementioned similarity observed in the results for each state. Also shown in Fig. 1 is the logarithm plot of the voting distribution using data from all candidates in Brazil competing for local representation. As in the case of São Paulo and other states of the

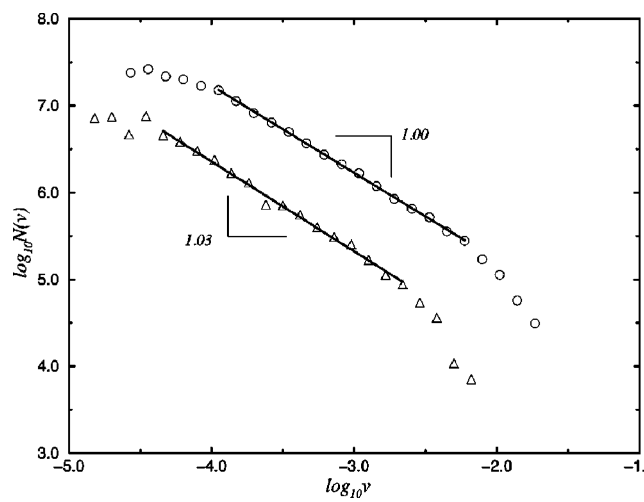


FIG. 1. Double logarithmic $\log_{10}$ plot of the voting distribution for São Paulo (triangle) and Brazil (circle). The solid lines are least-squares fits to the data in the scaling regions. The numbers indicate the scaling exponent $\alpha$.

 

nation, the data for Brazil can be adequately fitted to a decaying power law with an exponent very close to $-1$ ($\alpha = 1.00 \pm 0.02$).

One possible way to interpret the occurrence of the $1/v$ type of distribution that we find for proportional voting in Brazil is to assume that the success of the candidates determining $N(v)$ may be described by a typical *multiplicative process* [13]. Accordingly, the voting fraction of a candidate, $v$, can be viewed as a ''grand process'' depending on the successful completion of a number $n$ of independent ''subprocesses.'' Of course, these factors should be intrinsically related to the attributes and/or abilities of the candidate to persuade voters and obtain votes more effectively. As a result, one could then associate with each candidate the probability $p_i$ of performing the subprocess $i$ among voters, so that his or her voting fraction would be $v = p_1 p_2 \cdots p_n$. Assuming that $p_i$ are independent positive random variables and $n$ is sufficiently large, we readily obtain from the central limit theorem that the distribution of $v$ should be approximately *log normal*. Furthermore and more importantly, if the dispersion of the log-normal distribution is large enough, one can observe a hyperbolic profile, i.e., a $1/v$ type of distribution, over a wide range of random variable values [13]. Obviously, corrections should be expected at both small and large values of $v$ for the distribution to be normalizable and yet displaying the $1/v$*-like* profile at intermediate values of $v$ [14].

In Fig. 2, we show the cumulative distribution of $\log v$ for local representation in Brazil plotted on probability paper. On this type of semilogarithmic representation, a log-normal distribution should behave as a straight line with a slope which is simply its standard deviation. Clearly, the data shown in Fig. 2 indicate that approximately 90% of the candidates' voting follows a log-normal distribution with a standard deviation $\sigma \approx 2$.

Finally, it is possible to estimate the number $\ell$ of ''$e$-foldings'' for which a specified log-normal distribution mimics the $1/v$ type of behavior within a relative error $\theta$. According to West and Shlesinger [13], $\ell \approx 4\theta\sigma^2 + 1$. Taking $\theta = 0.2$, we obtain $\ell \approx 5$, which is equivalent to two orders of magnitude. This estimate is compatible with the results shown in Fig. 1 for the voting distributions of São Paulo and Brazil.

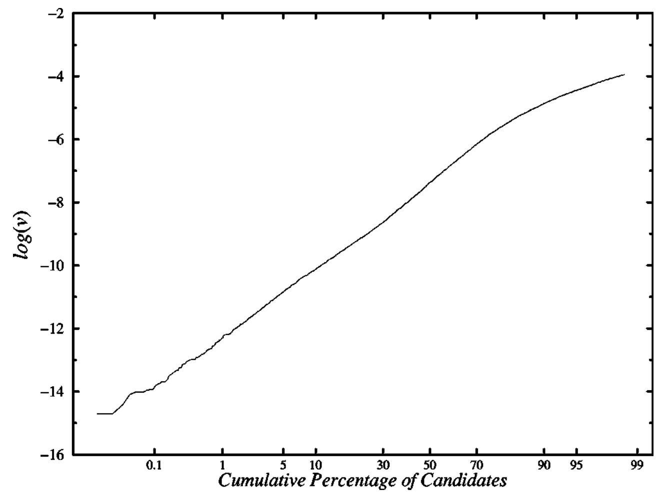


FIG. 2. Cumulative distribution of logarithm of the voting in Brazil plotted on probability paper.

As a conclusion, by analyzing the results obtained from the 1998 Brazilian proportional elections, we can state that a complex decision-making process, such as the candidate voting, can be explained in terms of a multiplicative mechanism. The similarity of results from all states gives strong evidence that the voting process is statistically regular for the whole country. This can be an indication that the voting motivations are similar for different states, regardless of the diversity in the number of voters, economic disparities, and local peculiarities. Brazil has the world's largest electorate subject to compulsory voting. This, together with the dissemination of information, mainly through television, can have a significant effect on the political composition of the electorate. It would be interesting to see whether or not these conjectures are also applicable to other countries and if they are dependent on the form of voting.

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